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Tunability of a Metal Plate Circular Polarizer in the Microwave Region

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Abstract- This paper discusses the tunability of a waveplate constructed using metal plate optics. Such plates can be used as optical retarding elements at single frequencies in the microwave region of the optical spectrum. In the X-band (3 cm. wavelength) the physical dimensions allow especially simple constructions.

Index Terms- microwaves, retarders, quarter-wave plate, plate optics

1. Introduction

Circularly polarized microwave radiation has gained attention partly because of its similarity to white light. In this vein it has attracted attention in communications, cellular telephones for example, because it allows the receiver to be less position sensitive. Because it reflects off of objects with less shadowing, there are fewer communication dead spots. The total extinction of circularly polarized radiation passing through a quarter wave plate of opposite handedness is a sensitive test for asymmetrical scattering since these scatterers convert the polarization to elliptical which is then detected. Consequently, ellipsometry using circularly polarized radiation in the microwave region shows potential as a diagnostic for sols and suspensions, plastic sheets as well as for dielectric composites including wood-based materials. Non-contact measurements of the properties of metals, semiconductors, the earth as well as the non-destructive evaluation of thick coatings through ellipsometry do not exhaust the list of possible applications [1-5]. In studies of the scattering of microwaves from objects, the Mueller scattering matrix elements of the scattering objects are measured. Measuring the Mueller scattering elements requires compensators and quarter wave plates. Thus, an economical, portable, and simple design for a quarter wave plate in the microwave region is needed. To best knowledge of the authors, no such device exists.

Most commonly, circularly polarized microwave radiation is produced using specially modified rectangular or circular waveguides. In some cases coaxial delay lines are used. While these methods can be very efficient, they are bulky and do not allow for the same degree of interchangeability of components as is common with visual optics. When performing measurements on materials, it may be necessary to have interchangeable wave plates that can placed between a horn and the specimen. When specimens are large, these components also need to be large. This need is incompatible with methods that require the polarizing optical elements to be placed inside a waveguide feeding a horn. There is a dearth of such elements when simplicity and economy are important. For example, sapphire wave plates resemble their optical counterparts and may be useful in the millimeter wave region of the spectrum. In the centimeter wave portion of the spectrum, they become unreasonably expensive.

Parallel plates in various ways have been used as optical elements. The practice of using parallel metal plates to make lenses dates from the early radar era [6,7]. A design for a quarter wave plate using free-standing metal plates was mentioned by van Vliet and De Graauw [8]. Some solid materials are optically anisotropic; their optical properties depend on the direction in which light propagates. This property is employed to construct various compensators in the optical region. This is the basis of the birefringence polarizer. Extensive discussion of this subject is given in various books; see for example the monograph of Hecht [9]. Here circularly polarized radiation in the microwave region is produced using a birefringent composite of metal plates and dielectric. Metal plate optics was used to make one type of lens by taking advantage of the fact that a wave could be guided by parallel metal plates if the electric vector was parallel to the plates. A different type of lens was produced when the magnetic vector was parallel with the plates. The phase velocity in these two types of lenses is significantly different. This suggests a way of making a birefringent composite using metal plates. In this composite the dielectric material both alters the phase velocity and provides mechanical support.

2. Description of the Circular Polarizer (X-band Quarter Wave plate)

We have constructed quarter wave plates by partially embedding metal slats in a dielectric material as shown in Fig. 1. In addition, Fig. 2 shows in detail two parallel plates only. The dielectric material partially fills the space between the plates to a height e. The width and the height each metal plate are d, and h, respectively, and the distance between the plates is a. Apart from the dielectric this diagram could describe a parallel plate guide of dimensions *a* by *h*. The dimension *h* can be arbitrarily large.

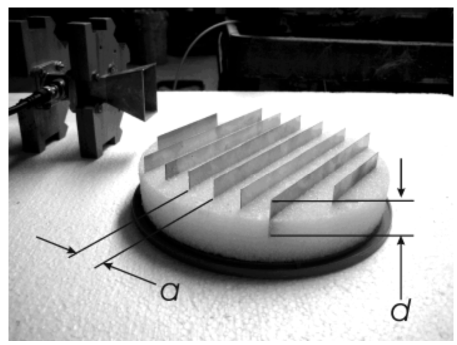

Fig. 1, X-Band Quarter wave plate (10.5 GHz).

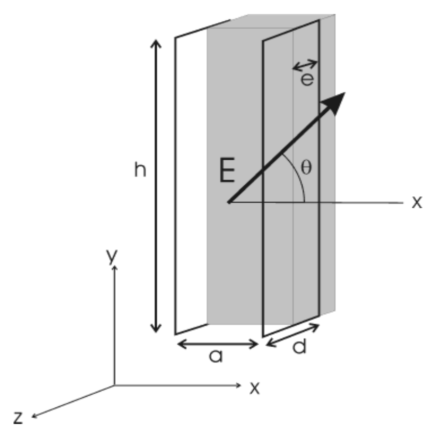

Fig. 2, Two Slats Separated by a Dielectric Block

A plane wave propagates in the +z-direction with its E-vector making an arbitrary angle θ with the x-axis as shown.

The incident electric field is in the x-y plane. The incident wave is resolved into two perpendicular component waves. One component is in the y-direction and the other one is in the x-direction. The wave whose E-field is parallel to the y-axis will be guided, while the one whose electric vector is parallel to the x-axis will be unguided. It is clear that, apart from incident reflection effects, the amplitude of the guided wave is the same as that of the unguided if the incident electric E makes an angle of 45º with the x-axis. Since the two above mentioned waves propagate at different speeds, the two waves recombine with elliptical polarization. If the two emerging components are of equal amplitude and the phase difference between them is π/2, the radiation will be circularly polarized. The phase difference may be adjusted by varying the dimension *e*. Sliding the slat deeper into the dielectric effectively shortens the path length of the guided wave without altering that of the unguided wave. With a suitable choice of dielectric and a sliding mechanism, a variable compensator can be constructed

3. Expressions of the Guided and Unguided Wavelengths

The described array of the parallel plate waveguides serves as a birefringent medium that circularly polarizes microwave radiation when the proper parameter values are chosen. Thus, some key equations of the rectangular waveguide are presented and discussed here to understand the operation of our quarter wave plate. At the risk of being redundant, we will repeat the development given in our previous description but in more detail [10].

The field equations are obtained by solving Maxwell’s equations for rectangular tubing [11,12]. Within the tubing the charge and electric current density are zero. Hence, the electric field **E** must satisfy the wave equation,

$$\nabla^2 \mathbf{E} = \mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}, \qquad (1)$$

where ε and μ are the permittivity the permeability of the medium through which the wave propagates. We write the time-dependence of **E** in the customary form

$$\mathbf{E}(r,t) = \mathrm{Re}\left(\mathbf{E}(r)\, e^{j\omega t}\right), \qquad (2)$$

where ω is the angular frequency in radians per second.

This allows us to write the wave equation for **E** as

$$\nabla^2 \mathbf{E} = -\omega^2 \mu\varepsilon\, \mathbf{E}. \qquad (3)$$

The magnetic field, **H,** also satisfies the above equation. These equations are solved simultaneously for the waveguide boundary conditions: The tangential component of **E** is zero, and the normal component of **H** is zero. The wave is thus confined to the waveguide but free to travel in the z-direction. Keeping the time-dependence separate, we allow the longitudinal variation, that along the axis of the guide, the z-axis, to be represented using the form

$$E(x,y,z) = E(x,y)e^{-\gamma z}, \qquad (4)$$

where $\gamma = \alpha + j\beta$, and is called the propagation constant.

Solutions for Eq. (3) in the form suggested by Eq. (4) are tabulated in textbooks [11] and are of two general types, transverse electric (TE) and transverse magnetic (TM). Both the electric and magnetic vectors, however, may not be transverse at the same time in a metallic waveguide. For each type of solution, there is, in principle, an infinite number of modes, eg. $TM_{1,2}$. However, the lowest order modes are those which allow the propagation of the lowest frequency waves and have the smallest cross-sectional dimensions for any given frequency. The lowest order transverse electric modes are $TE_{1,0}$ and $TE_{0,1}$ for rectangular waveguides. These have the lowest cutoff frequencies and are the modes most often used because the waveguide can be sized for single-mode operation that is usually desirable. The lowest order transverse magnetic modes are written analogously. For either the transverse electric or magnetic in rectangular waveguides the propagation constant γ is written,

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon} \quad . \qquad (5)$$

From Eq. (4) we see that γ must be pure imaginary if the wave is to propagate. In Eq. (5) *a* and *b* are height and width of a rectangular waveguide as we discussed earlier. The values *m* and *n* are called the mode numbers. Since γ is imaginary, then γ is also the wave number in imaginary units,

$$\gamma = j\beta = j\frac{2\pi}{\lambda_g} = j\frac{\omega}{c}, \qquad (6)$$

where *c* is the phase velocity in the guide and λ_g is the wavelength in the waveguide. It is easy to see from Eq. (5) and Eq. (6) if *m* and *n* are not equal and *a* and *b* are not equal, then swapping m and n will alter λ_g, the wavelength in the guide. We may consider two modes at the same frequency, eg. $TE_{1,0}$ and $TE_{0,1}$ having waves of equal amplitude propagating at the same time in a waveguide. These two waves will have different wavelengths in the waveguide if *a* and *b* are unequal. If, on account of this difference, the phase shift on passing through the guide amounts to a quarter wavelength, then a circularly polarized wave will be generated if the amplitudes of the component waves are equal. Depending on the relative lead or lag, right- or left-handed circular polarization will result.

While any non-square waveguide (four metallic sides) can be used to generate a circularly-polarized wave, the most compact approach is to have one of the two waves guided and the other propagating freely. Consider Eq .(5) for TE waves and allow *b* to become very large. For the 1,0 mode,

$$\beta_{1,0} = \sqrt{\omega^2\mu\varepsilon - \left(\frac{\pi}{a}\right)^2} \qquad (7)$$

and is independent of *b* in any case. For the 0,1 mode as *b* goes to infinity,

$$\beta_{0,1} = \omega\sqrt{\mu\varepsilon}\ . \qquad (8)$$

This is recognizable as the condition for an unguided wave. Thus, in the case of parallel metal plates forming a two-sided guide, *b* is infinity and a TE wave with its electric vector in the plane of the plates will be guided. A TE wave with its electric vector perpendicular to the plates will not be guided. The phase velocity of the guided wave is greater than that of the unguided wave.

It is possible to simplify Eq. (7) to one in terms of the wavelength within the rectangular guide using the free-space wavelength, λ_0,

$$\lambda_{1,0} = \frac{\lambda_0}{\sqrt{\varepsilon_r - \left(\frac{\lambda_0}{2a}\right)^2}}, \qquad (9)$$

where the dielectric constant $\varepsilon_r = \varepsilon / \varepsilon_0$. ε is the permittivity of the medium within the guide and ε_0 is the permittivity of vacuum. Following Eq. (8), for the unguided wave,

$$\lambda_{0,1} = \frac{\lambda_0}{\sqrt{\varepsilon_r}} \ . \tag{10}$$

An index of refraction (n) can be defined using the relation, $n = \lambda_0 / \lambda$. This can be applied to both guided and unguided waves.

4. Path Difference between the Orthogonal Components

Consider a plane wave propagating in the +z-direction with its E-vector making an angle θ with the x-axis as shown in Figs. 2 and 3. This wave is a superposition of two components with perpendicular E-vectors as diagrammed in Fig. 3.

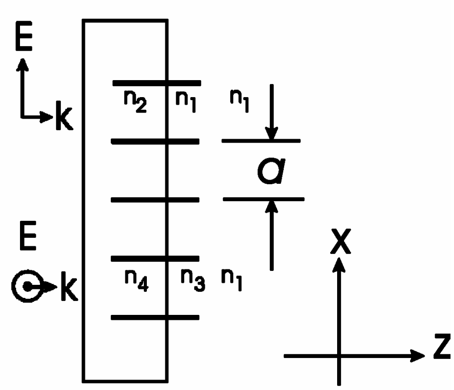

Fig. 3 an X-Z Cross Section of Wave Plate

The plates are viewed on end. The rectangular part of the diagram represents the dielectric.
The plates are arranged like slats in Venetian blinds separated by a distance *a*. In Fig 3. the slats extend into the page. Two vector **E**-**k** relations are shown at the left where **k** is the wave vector and points in the direction of propagation. The upper relation is for the unguided wave having wavelength $\lambda_{0,1}$. The lower relation is for the guided wave having wavelength $\lambda_{1,0}$. Since there are two waves and two dielectric constants, there are four indices of refraction and two general regions of propagation that effect the phase shift. For the unguided wave, $n_1 = 1$ in the free-space region and $n_2 = \lambda_0 / \lambda_{0,1}$ within the dielectric region. For the guided wave, the values of n_3 and n_4 are obtained analogously from Eq. (9) using the relative dielectric constant for free space (unity) and that for the dielectric material respectively.

The two waves undergo a relative phase shift from traveling through media with different indices of refraction. We desire to set the width of the slats, *d* in Fig. 2 and their projected dimension in Fig. 3, so as to obtain a relative phase shift of π/2 or some odd multiple of that. For the X-band, the physical dimensions are such that odd multiples would result in unnecessarily large dimensions. In any given region the relative phase shift is given by,

$$\Delta\phi = 2\pi \ d \ \frac{n_f - n_g}{\lambda_0} \ , \tag{11}$$

where n_f is the index of refraction for the free or unguided wave and n_g is that for the guided wave. These phase shifts are cumulative. If we set the total shift to π/2 and position the slats so that half their width is embedded in dielectric,

$$d = \frac{\lambda_0}{2\left(n_1 + n_2 - n_3 - n_4\right)} \quad . \qquad (12)$$

5. Tunability and Alignment

There are a total of three relevant interfaces at which reflections will occur. Since the indices of refraction are different for the two waves, the transmittance will not be the same for them. The total power transmittance for each of the two waves is readily calculated using Fresnel's equations. For circular polarization, the two component waves must be of equal intensity when they emerge from the plate rather than when they enter it. This is accomplished when the orientation angle of the slats with respect to the electric vector is slightly different from 45 degrees and is given by

$$\theta = \tan^{-1}\left[\left(\frac{T_f}{T_g}\right)^{1/2}\right], \qquad (13)$$

where T_f is the power transmittance for the non-guided wave, and T_g is the power transmittance for the guided wave. In practice, θ differed from 45˚ by a little over one degree.

Tunability can be defined as the ratio of the range of tuning (in GHz.) divided by the mean or design frequency in GHz. A theoretical value of 30% is typical and amounts to ±15%. Tuning is accomplished by sliding the slats in or out of the dielectric. Unless an exterior positioning frame is used, some of the slat must always remain in the dielectric. This reduces the tuning range.

Examining the plate's behavior using its Mueller matrix is helpful to understanding the process of alignment. For the purpose of analysis, we will assume that the plate is fixed with its fast axis (the guided axis) vertical. We will then rotate the incoming wave so that its electric vector makes and angle θ (see Fig. 1) with the normal to the slats. The Stokes vector for such a wave is:

$$\mathbf{S}(\theta) = \begin{vmatrix} \left\langle E_x^2 + E_y^2 \right\rangle \\ \left\langle E_x^2 - E_y^2 \right\rangle \\ 2\left\langle E_x^2\, E_y^2 \cos(\delta_0)\right\rangle \\ 2\left\langle E_x^2\, E_y^2 \sin(\delta_0)\right\rangle \end{vmatrix} = \begin{vmatrix} 1 \\ \cos(2\theta) \\ \sin(2\theta)\cos(\delta_0) \\ \sin(2\theta)\sin(\delta_0) \end{vmatrix}, \qquad (14)$$

where δ_0 is the phase lead of E_y with respect to E_x. We operate on this vector with the Mueller matrix representing the wave plate.

$$\mathbf{A} = \begin{vmatrix} (T_x + T_y)/2 & 0 & 0 & 0 \\ 0 & (T_x - T_y)/2 & 0 & 0 \\ 0 & 0 & (T_xT_y)^{\frac{1}{2}}\cos(\delta) & (T_xT_y)^{\frac{1}{2}}\sin(\delta) \\ 0 & 0 & (T_xT_y)^{\frac{1}{2}}\sin(\delta) & (T_xT_y)^{\frac{1}{2}}\cos(\delta) \end{vmatrix}, \qquad (15)$$

where δ is the phase delay introduced by the wave plate and is defined as before. Note that the fast axis is still the y-axis. The out going Stokes vector, **C**, is given by the vector product.

$$\mathbf{AS}(\theta) = \begin{vmatrix} T_x\cos^2(\theta) + T_y\sin^2(\theta) \\ T_x\cos^2(\theta) - T_y\sin^2(\theta) \\ (T_x\, T_y)^{\frac{1}{2}}\sin(2\theta)\cos(\delta_0 + \delta) \\ (T_x\, T_y)^{\frac{1}{2}}\sin(2\theta)\sin(\delta_0 + \delta) \end{vmatrix} = \mathbf{C}(\theta, \delta) \qquad (16)$$

From this, we deduce that $\delta_0 + \delta$ is entirely independent of θ and that the two adjustments must be made separately. As said before δ is fine-adjusted by sliding the slats into or out of the dielectric block.

While for any practicable spacing, *a*, there is a corresponding slat width *d*, the choice of *a* is not entirely arbitrary. If a is made too large, large secondary maximum lobes will appear in the guided component's transmitted radiation. These can be at least qualitatively understood by assuming a radiating element in the center between each pair of slats. If this element is assumed to radiate with a pattern defined by the Fresnel obliquity factor, this factor may be inserted into the standard antenna array formula to obtain a guess at the radiation pattern. Normalizing for the number of radiators gives,

$$F(\phi) = (1 + \cos(\phi)) \frac{\sin\left((m-1)\frac{\pi\, a}{\lambda_0}\ \sin(\phi)\right)}{(m-1)\sin\left(\frac{\pi\, a}{\lambda_0}\sin(\phi)\right)}, \qquad (17)$$

where *F* is the normalized amplitude, *m* is the number slats and ϕ is the azimuthal or scattering angle angle measured in the (tilted) plane perpendicular to the axes of the slats. It equals zero parallel to the z-axis as defined in Figs. 2 and 3. Plotting Eq. (17) shows that when *a* is less than 0.9 wavelength, side lobes do not reach the same magnitude as that of the main beam. For *m* in the range of 8 to 21 and *a* in the range of 0.4 to 0.7 times the wavelength, the amplitude of the side lobes is about 20% that of the main beam. The peak intensities of the side lobes are therefore on the order of 4% of the main beam. Experimentally, they have not been found to be as objectionable as this calculation might indicate[1].

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