

# A Quarter-wave Plate using Metal Plate Optics

Lawrence R. Lawson<sup>1\*</sup> and Hashim A. Yousif<sup>2</sup>,

<sup>1</sup>Wood Study Group P.O. Box 286 Bradford, PA 16701  
Tel: 814 368-7181; E-mail: me@larrylawson.net

<sup>2</sup>Physical and Computational Sciences Division  
University of Pittsburgh at Bradford  
Bradford, PA 16701, USA

Tel: 814 362-7603; E-mail: yousif@pitt.edu

**Abstract-** This paper discusses the construction of a quarter-wave plate as a retarding element for narrow-band use in the x-band region of the microwave spectrum. This retarder is constructed simply of aluminum plates embedded in a foam dielectric and is used with a horn radiator as an element in an ellipsometer. When a pair of these plates is used to generate and block circular-polarized radiation, extinction is good.

**Index Terms-** microwave ellipsometry, wave plate, metal plate optics, retarder, circular polarization, birefringence.

## I. INTRODUCTION

Microwave ellipsometry has been useful for studying the properties of a variety of materials and to the authors for studying the fiber orientations of wood. In this work it is convenient to have the analyzer quarter-wave plate external to the waveguide for two reasons. First, internal retarding elements require rotating joints which are complicated, likely to cause unwanted reflections and usually require cylindrical waveguides which do not reinforce polarization as do rectangular guides. Second, an external quarter-wave plate is easy to remove. A metal-plate type retarder used in conjunction with a horn radiator offers these advantages. Also, plate-type retarding elements are appealing because of their extreme simplicity. The practice of using parallel metal plates to make microwave lenses dates from the early radar era [1,2]. However the literature on plate-type retarders is scant. The possibility of making a retarding

element using free-standing metal plates was mentioned by van Vliet and De Graauw [3].

We have constructed a quarter wave plate by partially embedding metal slats in a dielectric material as shown in Figs.1 and 2.

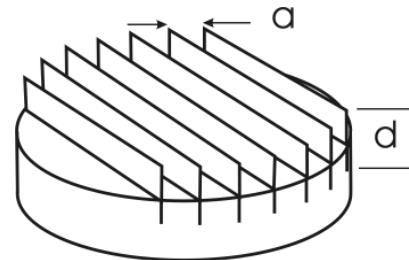


Fig. 1, Quarter-wave plate

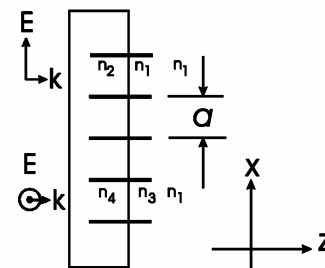


Fig. 2, An X-Z cross section of wave plate  
E indicated the direction of electric polarization; k indicates the direction of propagation.

The width and the height each metal plate are  $d$ , and  $h$ , respectively, and the distance between the plates is  $a$ . The dielectric material partially fills the space between the plates to a height  $e$ . The dimension  $h$  can be arbitrarily large. Embedding only a portion of the plates in a dielectric slab provides support for the plates also makes retardance of the assembly adjustable.

As a quarter-wave plate, a plane wave propagates in the  $+z$ -direction through it with its E-vector making an arbitrary angle of nearly  $\pi/4$  with the plates. ("Nearly" because there is a slight difference in reflectivity between the guided and unguided portions of the wave.) The incident wave is resolved into two perpendicular component waves. One component is parallel with the plates and the other one is perpendicular to them. The wave whose electric vector is parallel to the plates will be guided, while the one whose electric vector is perpendicular to the plates will be unguided. The amplitude of the guided wave is the same as that of the unguided wave. Since the two waves propagate at different speeds causing a relative delay of  $\pi/2$ , the two waves recombine with circular polarization. The phase difference may be adjusted by varying the dimension  $e$ .

To understand the operation of the metal-plate retarder, we may consider two modes at the same frequency, eg.  $TE_{1,0}$  and  $TE_{0,1}$  having waves of equal amplitude propagating at the same time in a waveguide. For the 1,0 mode, the propagation constant is

$$\beta_{1,0} = \sqrt{\omega^2 \mu \epsilon - \left(\frac{\pi}{a}\right)^2}. \quad (1)$$

Since the slats are not closed by a metallic boundary at the end, for the 0,1 mode the dimension corresponding to  $a$  goes to infinity,

$$\beta_{0,1} = \omega \sqrt{\mu \epsilon}. \quad (2)$$

This is recognizable as the condition for an unguided wave. The phase velocity of the guided wave is greater than that of the unguided wave.

It is possible to simplify (1) to one in terms of the wavelength within the rectangular guide using the free-space wavelength,  $\lambda_0$ ,

$$\lambda_{1,0} = \frac{\lambda_0}{\sqrt{\epsilon_r - \left(\frac{\lambda_0}{2a}\right)^2}}, \quad (3)$$

where the dielectric constant  $\epsilon_r = \epsilon/\epsilon_0$ .  $\epsilon$  is the permittivity of the medium within the guide and  $\epsilon_0$  is the permittivity of vacuum. Following (2), for the unguided wave,

$$\lambda_{0,1} = \frac{\lambda_0}{\sqrt{\epsilon_r}}. \quad (4)$$

An index of refraction,  $n$ , can be defined using the relation,  $n = \lambda_0/\lambda$ . This can be applied to both guided and unguided waves.

In or out of the dielectric the relative phase shift is given by,

$$\Delta\phi = 2\pi d \frac{n_f - n_g}{\lambda_0}, \quad (5)$$

where  $n_f$  is the index of refraction for the free or unguided wave and  $n_g$  is that for the guided wave. These phase shifts are cumulative. If we set the total shift to  $\pi/2$  and position the slats so that half their width is embedded in dielectric, referring to Fig. 2,

$$d = \frac{\lambda_0}{2(n_1 + n_2 - n_3 - n_4)}. \quad (6)$$

## II. EXPERIMENTAL

A pair of slat retarders were constructed as quarter-wave plates using styrofoam ( $\epsilon_r \approx 1.2$ ) as the dielectric. Slats were 2.81 cm. wide ( $d$ ) and spaced a distance of 2.04 cm. ( $a$ ) apart. They were with 9.5 cm. x 7.5 cm radiator and detector horns operating at 10.5 GHz. The detector was located 1 m from the source. Both the source and the detector are linearly polarized. The detector was a diode biased to be a linear square law detector.

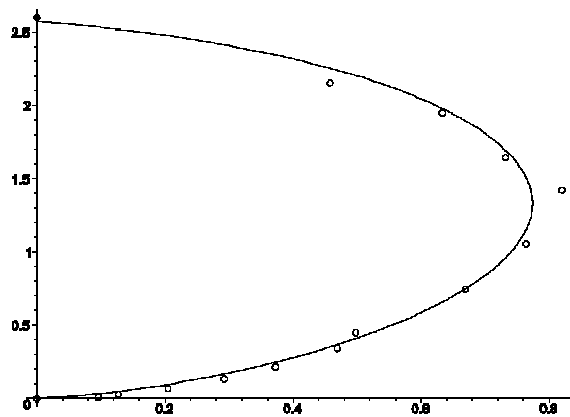


Fig. 3, Polar extinction curve for two quarter-wave plates

Both the abscissa and the ordinate represent detector volts. Circles indicate data points. The line is for reference only.

Fig. 3 shows the detector output vs. angle of rotation of the plate used as the analyzer. At zero degrees of rotation the output of the detector was zero. As the analyzer plate was rotated through  $\pi$  radians, a maximum output was reached as shown in Fig. 3. Fig. 3 deviated from that law of Malus. This deviation is attributed to a standing wave resulting from reflection from both the source and receiver and appears to be a characteristic of ellipsometers using horn-type detectors. Where the loss in sensitivity and increased noise can be tolerated, a non-reflecting receiver eliminates this annoyance.

## III. CONCLUSION

We have constructed and tested a quarter-wave retarding plate of a simple and apparently novel design primarily for use in a microwave ellipsometer. In the process of testing pairs of these plates, the authors observed standing wave effects which alter the shape of the extinction curve, shown in Fig. 3. These effects are not related to the type of quarter-wave plate but rather to the long wavelengths and high reflectivities of the horn transducers used. They are the subject of current study by the authors. The simplicity of the wave plate's design and its ease of construction suggests its potential application as a teaching aid as well as in simple devices to automatically determine grain orientation in wood.

## ACKNOWLEDGMENT

The authors acknowledge the assistance provided by Bradford Forest Products Inc.

## REFERENCES

- [1] A.H.F. van Vliet, and T. De Graauw, "Quarter Wave Plates for Submillimeter Wavelengths", *Int. J. Infrared and Millimeter Waves*, Vol. 2, 465-476, 1981.
- [2] Kock, W. E., "Path-Length Microwave Lenses", *Proc. I.R.E.*, Vol. 37, 852-855, 1949.
- [3] J. Ruze, "Wide-Angle Metal Plate Optics", *Proc. I.R.E.*, Vol. 38, 53-59, 1950.